Engineering Notes

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Calculation of Lift-Curve Slope Using a Wing Tip Biased Vortex Distribution

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Introduction

THE lift-curve slope of a finite wing C_{Lx} can be straightforwardly computed by paneling its surface with a set of discrete, horseshoe-shaped vortices distributed along the quarter-chord line. In effect, the entire wing is divided into a number of elementary wings whose lifting characteristics are each modeled by a single horseshoe vortex. The strength of each vortex, and hence the total lift, is calculated by simultaneously applying a zero normal flow condition at an equal number of control points that have been distributed along the wing's three-quarter-chord line. This method is basically a discretization of extended lifting-line theory, and the work of Blackwell[1] is a good example of its application to various problems.

Although this computational method is very powerful in its ability to determine the lift of a wing and systems of wings, it can require a large number of panels for an accurate solution, and more importantly, the more qualitative lift-reducing and drag-creating effects of the wing tips, which distinguish the finite wing from its two-dimensional counterpart, are not explicitly reflected in the geometry of the vortex system. Only by solving for the complete circulation distribution do the effects of the wing tips become apparent, since they are only implicitly represented in the structure of the model. An alternative horseshoe vortex-paneling scheme has been developed whose main property is that it models a wing as an indivisible structure whose tips explicitly differentiate its aerodynamic behavior from that of a two-dimensional airfoil. From another point of view, this means that the wing is not modeled as a collection of elemental wings whose distinguishing feature is their relative distances from the bulk of their neighbors.

Geometry of the Wing Tip Biased Vortex System

An exploded view of the vortex-paneling method is shown in Fig. 1, and Fig. 2 compares an overhead view of the wing tip biased system to a uniformly distributed vortex system. Essentially, the model consists of three parts: a core horseshoe vortex whose bound segment lies along the quarter-chord line and two inward cascades of vortices emanating from each wing tip that are stacked directly on top of the core vortex. The vortices in the wing tip cascades circulate in a direction opposite to that of the core vortex, and therefore explicitly cause a reduction in the overall lift of the wing. The width of the bound vortex segment of each component in a cascade is equal to one-quarter the width of the bound portion of the preceding horseshoe vortex, and the bound portion of each vortex lies along the quarter-chord of the wing. The control point associated with each horseshoe, which is required for the application of the normal flow boundary condition, is placed on the three-quarter-chord line halfway between its infinitely extending free vortex arms. In total, there are seven horseshoe vortices in the model and, a core vortex that is acted upon by two pairs of wing tip cascades that are each composed of three elements. Also, the cascade geometry is similar to that of the Cantor sets described by Mandelbrot.[2]

The geometric structure of the model is based on two fundamental flow concepts, which were first put forward by Lanchester[3] and later refined by Prandtl,[4] that are generally accepted as accounting for the nature of the flow in the vicinity of a wing. On a global scale, as illustrated by Prandtl, the finite wing is aerodynamically similar to the core horseshoe vortex. However, on a local scale, the spanwise flow in the region of the wing tips is not adequately represented by the basic core vortex. The wing tips allow more fluid to circulate to the upper surface than can be accommodated by this model, and therefore the core vortex by itself would tend to not correctly predict the lift generated by the wing. Introduction of the wing tip cascades, as first described in qualitative terms by Lanchester, accounts for the more elaborate local spanwise distribution of circulation over the wing that is created by the tips. Classically, the variation in circulation along the wing is accounted for by allowing the vortex strength to be a continuous function of the spanwise coordinate. This naturally leads to discretized computational methods like that outlined in the introductory discussion. The present model is an attempt to shift the emphasis from viewing the finite wing as an aerodynamic structure composed of many geometrically similar elemental structures of varying strength to viewing it as a fundamentally two-dimensional aerodynamic structure that is explicitly modified by vortex creation at the wing tips.

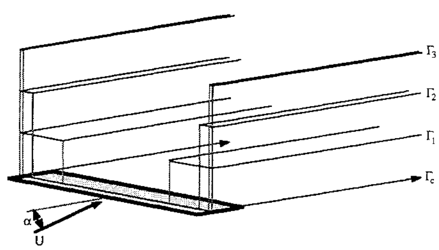

Fig. 1 Vertically exploded view of the horseshoe vortex stacking scheme.

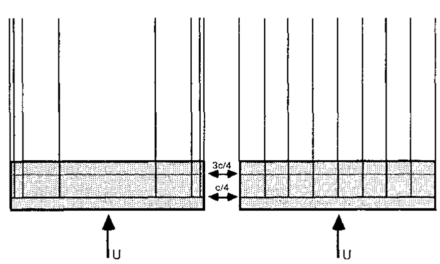

Fig. 2 A top view comparison of a wing tip biased vortex distribution with a uniform vortex distribution.

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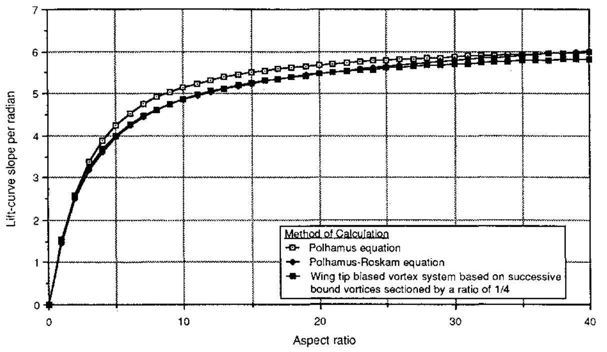

Fig. 3 Lift-curve slope of planar, untwisted rectangular wings calculated using a wing tip biased horseshoe vortex method with the wing tip cascades confined to the outer quarters of the wing.

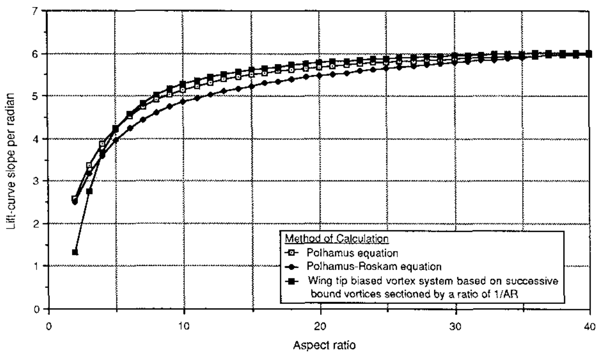

Fig. 4 Lift-curve slope of planar, untwisted rectangular wings calculated using a wing tip biased horseshoe vortex method with the wing tip cascades confined to the outer 1/AR sections of the wing.

Calculation of the Lift-Curve Slope

The mathematical methods used to calculate the induced velocity at each control point, to solve for the vortex strengths, and to integrate for the lift and, eventually, the lift-curve slope are the same as those used in conventional horseshoe vortex methods;[1] only the arrangement of the vortices is different. One further note: the wing tip vortex strengths, Γ_1, Γ_2, and Γ_3, do not need to have their direction of circulation set opposite to that of the core vortex at the outset. All vortices could have the same direction and the solution procedure would subsequently determine the proper circulation directions. This is to say that the directions of circulation need not be arbitrarily predetermined but arise naturally as a consequence of the model's geometry. Figure 3 plots $C_{L\alpha}$ for planar, untwisted rectangular wings calculated using the wing tip biased vortex system over a range of aspect ratios, AR, and compares it to values obtained from the standard Polhamus and Polhamus-Roskam equation.[5] The Polhamus-Roskam equation is a version of the Polhamus equation for $C_{L\alpha}$ that has been empirically modified to improve its agreement with experimental data. Figure 3 shows that there is good agreement between the $C_{L\alpha}$ calculated from the wing tip biased vortex system and the Polhamus-Roskam equation for aspect ratios under 30. For aspect ratios between 30 and 40, the wing tip biased method consistently underpredicts $C_{L\alpha}$. The difference between solutions is at most 3% at an aspect ratio of 40. Also, the convergence to the two-dimensional lift-curve slope at high aspect ratios is slower for the wing tip biased method.

The ratio of the lengths of the bound segments of successive vortices in each cascade can be set to 1/AR instead of 1/4, as in the original model, to improve the convergence properties at high aspect ratios. Figure 4 compares the values of $C_{L\alpha}$ calculated with this modification to those calculated from the Polhamus and Polhamus-Roskam equations. At high AR the convergence properties are improved, but for AR below 30 the agreement with either theory is poor. The 1/AR sectioning ratio is a good model of the limited wing tip vortical effects that are experienced by nearly aerodynamically two-dimensional, high AR wings because the cascades are confined to the outermost extremities of the wing. A model based on a sectioning ratio of 1/4 can approximate the vortical effects of the wing tips over a broader range of AR but tends to break down at high AR because the cascades extend too far into the center of the wing.

Conclusions

A horseshoe vortex model based on wing tip biased cascades of vortices modifying a central vortex core has been used to calculate the lift-curve slope of a flat, untwisted rectangular wing. If the cascades are confined to the outer quarters of the wing, the lift-curve slope can be calculated to a reasonable degree of accuracy over a broad range of aspect ratios using seven vortex elements. At very high aspect ratios greater than 30, limiting the wing tip cascades to the outermost 1/AR fractions of the wing improves the accuracy of the calculations. This provides a method of calculating aerodynamic properties that is not computation intensive while at the same time attempting to model the more qualitative properties of the flow.

Conventional horseshoe vortex methods based on extended lifting-line theory can be modified to model the rapid changes in flow properties near the wing tips by using nonuniform distributions of vorticity. One such technique is the so-called "circle distribution method" whereby a semicircle, whose diameter is the quarter-chord line, is erected over the wing and equally spaced arcs are projected onto the quarter-chord line to form the bound vortex segments. As a result, the horseshoe vortices are more densely packed near the tips than in the center of the wing. However, the concept of the whole wing as a composite of many smaller wings is not changed from the uniform distribution method; only the distribution of vorticity has changed. The main difference between the wing tip biased method and other methods is in its perception of the wing as a simple horseshoe-shaped vortex structure that is explicitly modified by its tips.

References

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Flutter Modes of High Aspect Ratio Tailless Aircraft

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IN an earlier Note,[1] the author investigated the symmetric flutter characteristics of a class of high aspect ratio, tailless aircraft (sailplanes) and compared the results with those of an existing tailed aircraft of similar class (the Kestrel-22m). A normal mode approach was sought so that the flutter determinant was formed by expressing the mass, stiffness, and aerodynamic matrix of the aircraft in terms of the generalized

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